Math 1B Quiz 1 Version 1
Wed Apr 19, 2017

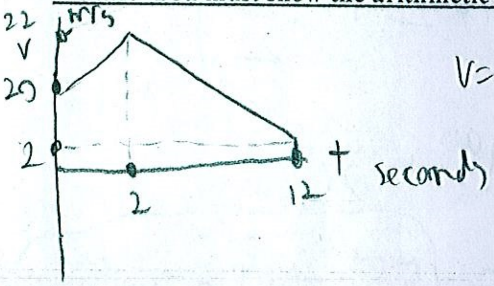
NAME YOU ASKED TO BE CALLED IN CLASS:

SCORE: _____ / 30 POINTS

1. **No calculators / notes / unauthorized paper, electronics or communication allowed**
2. **Simplify all answers unless stated otherwise**
3. **Show proper calculus level work to justify your answers**

A person's velocity (in meters per second) at time t (in seconds) is given by $v(t) = \begin{cases} 20+t, & 0 \le t \le 2 \\ 26-2t, & 2 \le t \le 12 \end{cases}$. **SCORE: <u>5</u> / 5 PTS**

a] Find the exact distance the person travelled from time $t=0$ seconds to $t=12$ seconds.
<u>NOTE: You must show the arithmetic expression that you used to get your answer.</u>



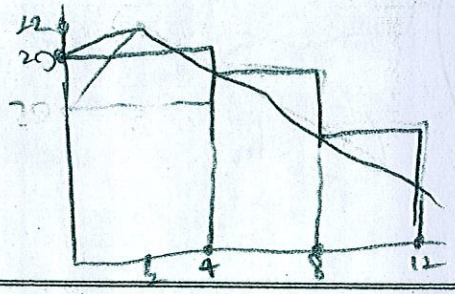
$V = A(t)$

$A_{trap} = \frac{b_1+b_2}{2} h$ ① $\frac{20+22}{2} \cdot 2 = 42$

$A_{trap} = \frac{b_1+b_2}{2} h =$ ① $\frac{22+2}{2} \cdot 10 = 120$

$120 + 42 = \boxed{162 \text{ meters}}$ ½

b] Estimate the distance the person travelled from time $t=0$ seconds to $t=12$ seconds using three subintervals and left endpoints.
<u>NOTE: You must show the arithmetic expression that you used to get your answer.</u>



$\Delta x = \frac{b-a}{n} = \frac{12-0}{3} = 4$

$A \approx \sum_{i=0}^{n-1} f(x_i)\Delta x \approx \sum_{i=0}^{2} f(x_i)\Delta x$

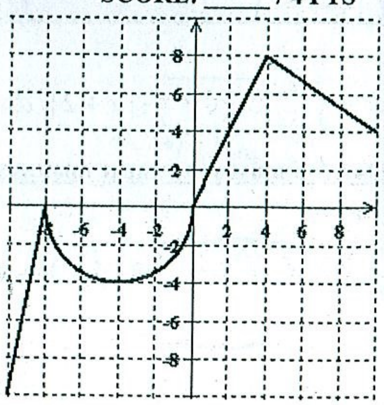
$A \approx f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x$

½ $\approx f(0)\cdot 4 + f(4)\cdot 4 + f(8)\cdot 4$

$\approx 20\cdot 4 + 18\cdot 4$ ½ $+ 10\cdot 4$ ½

$\boxed{A \approx 192 \text{ meters}}$ ①

The graph of function f is shown on the right. **SCORE: _____ / 4 PTS**
<u>The graph consists of a diagonal line, an arc of a circle, then two additional diagonal lines.</u>



a] Evaluate $\int_{-10}^{10} f(x)\,dx$.
<u>NOTE: You must show the arithmetic expression that you used to get your answer.</u>

$\int_{-10}^{-8} f(t)dt + \int_{-8}^{0} f(t)dt + \int_{0}^{4} f(t)dt + \int_{4}^{10} = \int_{-10}^{10} f(t)dt$

$-(\frac{1}{2}\cdot 16\,bh) + -(\frac{1}{2}\pi r^2) + \frac{1}{2} bh + \frac{(b_1+b_2)}{2} h$

$-10 - 8\pi + 16 + 36 = \boxed{42 - 8\pi = \int_{-10}^{10} f(t)dt}$ ½

b] Evaluate $\int_{10}^{-4} f(x)\,dx$.

① $\int_{10}^{-4} f(t)dt = -\int_{-4}^{10} f(t)dt = -\left(\int_{-4}^{0} f(t)dt + \int_{0}^{4} f(t)dt + \int_{4}^{10} f(t)dx\right.$

$= -\left(-\frac{1}{4}\pi r^2 + \frac{1}{2} bh + \frac{b_1+b_2}{2} h\right) = -(-4\pi + 52) = \boxed{4\pi - 52 = \int_{10}^{-4} f(t)dx}$ ½

Using the limit definition of the definite integral, and right endpoints, find $\int_{-4}^{-1}(2x^2+8x)\,dx$. **SCORE:** 5 **/ 10 PTS**

NOTE: Solutions using any other method will earn 0 points.

$f(x) = \int_{-4}^{-1} (2x^2 + 8x)\,dx$

$= \lim_{n\to\infty} \sum_{i=1}^{n} (2x^2 + 8x)\Delta x$ 1

$= \lim_{n\to\infty} \left(\sum_{i=1}^{n} 2x^2 \Delta x + \sum_{i=1}^{n} 8x \Delta x \right)$ 1

$= \lim_{n\to\infty} \left(\sum_{i=1}^{n} 2\left(\frac{3i}{n}\right)^2 \cdot \frac{3}{n} + \sum_{i=1}^{n} 8 \cdot \frac{3i}{n} \cdot \frac{3}{n} \right)$

$= \lim_{n\to\infty} \left(\sum_{i=1}^{n} \frac{57i^2}{n^3} + \sum_{i=1}^{n} \frac{72i}{n^2} \right)$

$= \lim_{n\to\infty} \left(\frac{57}{n^3} \cdot \sum_{i=1}^{n} i^2 + \frac{72}{n^2} \sum_{i=1}^{n} i \right)$

$= \lim_{n\to\infty} \left(\frac{57}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} \right.$ 1 $\left. + \frac{72}{n^2} \cdot \frac{n(n+1)}{2} \right)$ 1

$\Delta x = \frac{b-a}{n} = \frac{-1-(-4)}{n} = \frac{3}{n}$

$\to \lim_{n\to\infty} \left(\frac{57}{6} \cdot \frac{n(n+1)(2n+1)}{n^3} + \frac{72}{2} \cdot \frac{n(n+1)}{n^2} \right)$

$= \left(\frac{57}{6} \cdot 2 + \frac{72}{2} \right)$

$= 19 + 36$

$= \boxed{55 = \int_{-4}^{-1} 2x^2 + 8x\,dx}$

1 $\lim_{n\to\infty}$

Evaluate $\int_{-6}^{0}(2\sqrt{36-x^2} - |x+2|)\,dx$ using the properties of definite integrals and interpreting in terms of area. **SCORE: _____ / 5 PTS**

cir rad 6

NOTE: You must show the proper use of the properties of the definite integral, NOT just the arithmetic.

$A = \int_{-6}^{0} (2\sqrt{36-x^2} - |x+2|)\,dx$

$A = \int_{-6}^{0} 2\sqrt{36-x^2}\,dx - \int_{-6}^{0} |x+2|\,dx$

$= 2\int_{-6}^{0} \sqrt{36-x^2}\,dx - \left(\int_{-6}^{-2} -(x+2)\,dx + \int_{-2}^{0} (x+2)\,dx \right)$

$= 2 \cdot \frac{1}{4}\pi 6^2$ 1 $- \left(\left(-\frac{1}{2} 4 \cdot 4\right) + \frac{1}{2} 2 \cdot 2 \right)$ 1

$= 18\pi - 10$

$\boxed{A = 18\pi - 10}$ 1